

More on tachyon cosmology in de Sitter gravity

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ABSTRACT: We aim to study rolling tachyon cosmological solutions in de Sitter gravity. The solutions are taken to be flat FRW type and these are not time-reversal symmetric. We find that cosmological constant of our universe has to be fine-tuned at the level of the action itself, as in KKLT string compactification. The rolling tachyon can give rise to required inflation with suitable choice of the initial conditions which include nonvanishing Hubble constant. We also determine an upper bound on the volume of the compactification manifold.

KEYWORDS: Cosmology of Theories beyond the SM, Tachyon Condensation.

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1. Introduction

Recently the phenomenon of tachyon condensation [1, 2] has been a subject of much attention in string theory as well as in cosmology [3, 4]. The tachyon field appears as an instability in non-supersymmetric p -branes dynamics as well as an instability in brane-anti-brane system in superstring theory. The low energy tachyonic field theory is governed by the Born-Infeld type action [5, 1, 2]

$$- \int d^4x V(T) \sqrt{-\det(g_{\mu\nu} + 2\pi\alpha' \partial_\mu T \partial_\nu T)} \quad (1.1)$$

where tachyon T appears explicitly as a world volume scalar field, $g_{\mu\nu}$ represents the pull-back of the spacetime metric and $V(T)$ is a positive definite tachyon potential. In the flat spacetime equation of motion of this action, for purely time-dependent tachyon, assume following unique form:

$$\frac{V(T)}{\sqrt{1 - 2\pi\alpha' \dot{T}^2}} = \rho. \quad (1.2)$$

It means that the tachyon field behaves like a fluid of constant positive energy density, ρ , and of negative pressure, $P = -V \sqrt{1 - 2\pi\alpha' \dot{T}^2}$. During the time evolution tachyon field rolls down the potential and reaches its true vacuum value $T = \infty$ where $V(\infty) = 0$, while keeping ρ constant. When this tachyon system is coupled to background supergravity this result will, however, get significantly modified and new type of solutions can be obtained. In recent time the applications of open string tachyon in inflationary cosmological models have been quite encouraging [6–15, 3, 4, 16].¹

¹Also a wide class of useful references on this topic can be found in [3, 4, 17].

In this paper we plan to study a very specific model in which tachyon field is coupled to de Sitter gravity. It will be assumed that compactification of string theory to four-dimensional de Sitter spacetime of KKLT type [18] can be achieved in which all the moduli including the volume modulus are fixed. Our aim is to study spatially flat, homogeneous cosmological solutions in this model. The solutions we discuss are completely *non-singular* and could describe inflationary situations just like after the big-bang and also at late time may possibly describe the universe as of today. However, it will require fine-tuning of the cosmological constant at the supergravity level itself. The paper is organised as follows. In section-2 and section-3 we work out the salient features of simple inflationary models which are obtained by coupling bulk supergravity to the tachyon action. In section-4, we consider the case where gravity with bulk cosmological constant is coupled to the tachyon action. We restrict only to small cosmological constant so that its effects on tachyon evolution are minimal. We estimate the number of e-folds during inflation. We also obtain an upper bound on the size of compactification manifold. Section-5 contains the numerical analysis where we can explicitly see the behaviour of various physical quantities. We summarise the main results in section-6.

2. De Sitter gravity with tachyon field

We follow the model considered by Sen [3] based on tachyon field theory coupled to background gravity and bulk cosmological constant, Λ . We assume that all moduli are frozen possibly as in KKLT scenario [18]. The four-dimensional effective action is taken as

$$S = \int d^4x \left[\frac{1}{16\pi G} \sqrt{-g}(R - 2\Lambda) - V(T) \sqrt{-\det(g_{\mu\nu} + \partial_\mu T \partial_\nu T)} \right] \quad (2.1)$$

where Newton's constant G is related to four-dimensional Planck mass M_p as $8\pi G = M_p^{-2}$. We have also taken $2\pi\alpha' = 1$ which sets string tension to unity. In this section our analysis will not explicitly depend upon the actual form of $V(T)$. But, we shall require that at the top of the potential, $V(T_0) = V_0$ is large but finite, while $V(\infty) = 0$ in the vacuum. These consist two well known properties of the tachyon potential, e.g. $V(T) = V_0/\cosh(T/\sqrt{2})$ in superstring theory [2]. Later on we shall take $\Lambda \ll GV_0$, because we would like to have solutions in our model to emerge with the properties of the universe as observed today, and also with this condition effect of Λ on tachyon rolling will be minimal. Motivated by experimental inputs, that universe appears *spatially flat* and homogeneous at large scales, we shall look for time-dependent but not necessarily time-reversal symmetric solutions of equations of motion of the action.² Hence we take the metric ansatz to be the Friedman-Robertson-Walker metric

$$ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right) \quad (2.2)$$

and the tachyon to be purely time-dependent $T = T(t)$. Although, k can have any value 0 or ± 1 but in this paper we shall be only discussing $k = 0$, i.e. spatially flat FRW case.

²The time-reversal symmetric solutions have been discussed by Sen [3]. The recent cosmological measurements (WMAP) could be found in [19].

With the above ansatz the field equations derived from the action can be written as, the Tachyon equation:

$$\ddot{T} = -(1 - \dot{T}^2) \left(\frac{V'}{V} + 3\dot{T} \frac{\dot{a}}{a} \right) \quad (2.3)$$

the Friedman equation:

$$\frac{\ddot{a}}{a} = \frac{\Lambda}{3} + \frac{8\pi G}{3} \left[\frac{V(T)}{\sqrt{1 - \dot{T}^2}} \left(1 - \frac{3}{2} \dot{T}^2 \right) \right] \quad (2.4)$$

and the Raychaudhuri equation:

$$\left(\frac{\dot{a}}{a} \right)^2 = \frac{\Lambda}{3} + \frac{8\pi G}{3} \frac{V(T)}{\sqrt{1 - \dot{T}^2}}. \quad (2.5)$$

It can be seen that the equation (2.3) follows simply by taking the time derivative of the eq. (2.5). Therefore equations (2.4) and (2.5) contain all information about tachyon rolling in this model, making (2.3) redundant. Let us introduce the quantity $H(t) \equiv \dot{a}(t)/a(t)$ known as Hubble constant at any given time t . In this simple model equations (2.4) and (2.5) immediately give us

$$\dot{H} = -\frac{3}{2} \frac{8\pi G}{3} \frac{V(T)}{\sqrt{1 - \dot{T}^2}} \dot{T}^2 \quad (2.6)$$

This is the key equations for the evolution of the Hubble constant. Since the quantity $\frac{V(T)}{\sqrt{1 - \dot{T}^2}}$ on the right hand side of the equation (2.6) is always positive definite, while \dot{T}^2 varying between 0 and 1 monotonically, this equation implies,

$$\frac{dH}{dt} \leq 0.$$

That is, *no matter where we are in tachyon evolution, Hubble constant always decreases with time or at most becomes constant, but it cannot increase in time.* We shall see explicitly in the following that $H(\infty)$ is either vanishing or assumes a constant value at late times.

The equations (2.5) and the (2.6) can further be assembled in the following useful form

$$\frac{\dot{H}}{H^2 - \lambda^2} = -\frac{3}{2} \dot{T}^2 \quad (2.7)$$

where $\lambda^2 \equiv \Lambda/3$. Note that from (2.5) only those situations are allowed for which $H(t) \geq \lambda$. This equation can be immediately integrated to know the actual time dependence of H provided we know how \dot{T} changes with time, for that one has to solve tachyon equation (2.3) exactly for the given potential V . Nevertheless many useful conclusions can still be drawn. We will consider two special cases where $\Lambda = 0$ and $\Lambda > 0$ in what follows.

3. Cosmology with $\Lambda = 0$

This case has rather been studied in quite detail in the literature [6–8, 10].³ We study this case here again to gain more insight. In fact in this particular case (2.7) reduces to a

³To collect all important references on this, however, one can see recent papers [3, 4] which include most of them.

very simple form

$$\frac{\dot{H}}{H^2} = -\frac{3}{2}\dot{T}^2 \quad \text{Or} \quad \frac{d}{dt}(H^{-1}) = \frac{3}{2}\dot{T}^2 \quad (3.1)$$

which can be integrated immediately to give *Hubble time* (the inverse of Hubble constant)

$$H^{-1}(t) = \frac{3}{2} \int_0^t \dot{T}^2(t') dt' + \frac{1}{H_0} \equiv F(t) \quad (3.2)$$

where H_0 is an integration constant which is positive definite and has to be fixed from initial conditions at $t = 0$. The usefulness of the above equation is evident in that it is an integration of the function $(\dot{T})^2$ over the time interval t elapsed after $t = 0$. Since \dot{T}^2 is a well behaved quantity and can only vary between 0 and 1, without knowing its exact form we can draw following conclusions;

(1) As $t \rightarrow \infty$, we will have $F(t) \rightarrow \infty$ and so $H(\infty) = 0$. Thus Hubble constant vanishes in future and no-matter-what universe becomes flat.

(2) Consider two different times t_1, t_2 ($t_1 < t_2$), from (3.2) the change in Hubble time in the interval $\Delta t = t_2 - t_1$ is given by

$$\Delta H^{-1} = H^{-1}(t_2) - H^{-1}(t_1) = \frac{3}{2} \int_{t_1}^{t_2} \dot{T}^2(t) dt \quad (3.3)$$

which is independent of the initial value H_0 . If \dot{T}^2 varies slowly in the short interval Δt then

$$\frac{\Delta H^{-1}}{\Delta t} \simeq \frac{3}{2} \dot{T}^2(t_1) \quad (3.4)$$

Thus by measuring the Hubble constant at two different instances will give us an idea of rate of change of tachyon field and vice-versa. Also since $\dot{T}^2 \leq 1$ this provides us with a bound, that for a short interval Δt

$$\frac{\Delta H^{-1}}{\Delta t} \leq \frac{3}{2}. \quad (3.5)$$

which can be easily tested. Now coming back, the Raychaudhuri equation (2.5) becomes

$$\frac{8\pi G}{3} \frac{V(T)}{\sqrt{1-\dot{T}^2}} = H^2 = \left(\frac{1}{F(t)} \right)^2 \quad (3.6)$$

We see that tachyon equation (2.3) is immediately satisfied by (3.6). Thus we have been able to reduce the problem to the first derivatives of the quantities involved. Now we can fix other things from here. Since $F(\infty) = \infty$ we learn from (3.6) $\frac{V(\infty)}{\sqrt{1-\dot{T}^2}} = 0$ at $t = \infty$. It means as time passes, $V \rightarrow 0$ faster than $\dot{T} \rightarrow 1$ so that $\frac{V(T)}{\sqrt{1-\dot{T}^2}} \rightarrow 0$. More precisely, it is evident from (3.2) that at late stage in the evolution $H(t) \propto \frac{1}{t}$ that implies

$$\text{Lim}_{t \rightarrow \infty} \frac{V(T)}{\sqrt{1-\dot{T}^2}} \propto \frac{1}{t^2}.$$

This is quite unlike in pure tachyon condensation where $V \rightarrow 0$ with $\dot{T} \rightarrow 1$, keeping $\frac{V(T)}{\sqrt{1-\dot{T}^2}}$ fixed, see eq. (1.2).

Now at $t = 0$, $T(0) = T_0 = 0$ and $V = V_0$, but what about the initial value of \dot{T} at $t = 0$? We find that $\dot{T}(0)$ can have any generic value between 0 and 1, but certainly it must be less than 1. It is plausible to take $\dot{T}(0) = 0$, with that

$$H(0) = H_0 = \sqrt{\frac{8\pi G}{3}V_0}, \tag{3.7}$$

which makes initial value of the Hubble constant tuned to the height of tachyon potential. Also this initial value has other important implications for inflation in our model. With the choice that $\dot{T}(0) = 0$, we immediately learn from eq. (3.1) that near the top of the potential

$$|\dot{H}| \ll H^2$$

automatically, which is the key requirement for inflation in the cosmological models.⁴ Also if we want the initial value H_0 to be large, only way we can do it is by increasing the height V_0 of the tachyon potential. Taking H_0 very large but finite, an initial situation we may describe as just after *big-bang*, where universe starts with an explosion.⁵ It should however be emphasized that our solutions are completely *non singular*. Note that we are dealing with an effective action and so initial big-bang singularity, if any, cannot be addressed in this model. For that one will have to work in full string theory.

In summary, we find from eqn (3.2) that in this $\Lambda = 0$ model not only \dot{H} vanishes in future but also H . That means effective cosmological constant vanishes at late time and universe becomes flat. Before closing this section let us also find out how does the tachyon evolve in its vacuum as $t \rightarrow \infty$. In the tachyon vacuum, $T \sim \infty$, $H \sim 0$ while \dot{T} is still less than 1. The tachyon equation effectively assumes the following form:

$$\ddot{T} \simeq (1 - \dot{T}^2) \left(\frac{1}{\sqrt{2}} \right) \tag{3.8}$$

where we have specifically taken the potential to be $V = V_0/\cosh(T/\sqrt{2})$. This equation suggests that late time evolution of tachyon is decoupled from gravity. The late time equation (3.8) has an exact solution $T \sim \sqrt{2}\ln \cosh(\frac{t}{\sqrt{2}})$. Thus ultimately $T \sim t$ as $t \rightarrow \infty$, i.e. tachyon field becomes time or in other words $\dot{T} \rightarrow 1$. This has been the most common feature in tachyon condensation [1, 2].

3.1 Matter dominated phase

We can also see universal behavior as $t \rightarrow \infty$. In the neighbourhood of $\dot{T} \approx 1$, from (3.1)

$$\frac{d}{dt}H^{-1} \approx \frac{3}{2}.$$

⁴To emphasize this inflationary condition directly follows when $\dot{T} \sim 0$ and has no bearing on the value of H . In fact inflation lasts so long as $\dot{T}^2 \ll 2/3$.

⁵However there are problems in taking V_0 large in string tachyon models, see [9]. To estimate, $GV_0 \sim \frac{Ng_s M_s^2}{v_0}$, where $g_s \ll 1$ is string coupling, $v_0 \equiv (R/l_s)^6$ is the dimensionless compactification volume parameter and N is the number of coincident D3-branes. When converted to four-dimensional Planck mass units, using the relation $M_s = \sqrt{\frac{g_s^2}{v_0}}M_p$, we find $GV_0 \sim \frac{Ng_s^3}{v_0^2}M_p^2$ is, in general, astronomically much larger than currently observed value of cosmological constant, $\Lambda_{observed} \approx 10^{-122}M_p^2$.

This tells us that in the far future the scale factor behaves as $a(t) \sim t^{2/3}$ and $H \sim \frac{2}{3} \frac{1}{t}$, which is characterised as the *matter dominated phase of universe* and is the decelerating one ($\ddot{a} < 0$) [20]. It means that the expanding universe enters into a decelerating matter dominated phase towards the end of evolution. The deceleration is maximum as we reach closer to the tachyon vacuum. Here it is useful to mention that current phase of our universe is dominantly a vacuum phase, where cosmological constant drives the expansion. On that basis the models with $\Lambda = 0$ perhaps can only describe the inflationary situations (just after big-bang) and may not physically describe the universe today. In the next section we will see that by allowing non-zero Λ in the action can change the situation.

4. Cosmology with $\Lambda \neq 0$

This is an important case to study and we shall assume a non-vanishing positive cosmological constant in the effective action (2.1), which can arise in string compactifications to four dimensions in very special situations, as in KKLT compactification [18]. Recently Sen has studied this type of tachyon cosmologies in [3]. The difference here is, since we will consider only $k = 0$ FRW cosmologies, we cannot start with the time-reversal symmetric initial conditions as in [3]. The crucial difference is the Hubble constant will never be zero in our model. We repeat the calculations of the last section but now including the finite cosmological constant Λ . Consider the equation (2.7)

$$\frac{\dot{H}}{H^2 - \lambda^2} = -\frac{3}{2}\dot{T}^2 \tag{4.1}$$

which after integration gives

$$H(t) = \lambda \coth[\lambda F(t)] \tag{4.2}$$

where

$$F(t) = \frac{3}{2} \int_0^t \dot{T}^2(t') dt' + H_0^{-1}$$

with H_0 being an integration constant which has to be fixed from initial conditions at $t = 0$. We shall consider those situations only in which $H(0) \gg \lambda$. Again the usefulness of the above equation is evident in the simplicity of the right hand side expression which involves area under the curve $\dot{T}^2(t)$ over the time interval t elapsed after $t = 0$. We draw the following conclusion.

Since for $t \rightarrow \infty$, $F(t) \rightarrow \infty$, hence $H(\infty) = \lambda$. Thus in this de-Sitter type model, Hubble constant becomes constant in future, no-matter how large initial value of H the universe starts with.

The equation (2.5) becomes

$$\frac{8\pi G}{3} \frac{V(T)}{\sqrt{1 - \dot{T}^2}} = \lambda^2 \operatorname{cosech}^2[\lambda F(t)] \tag{4.3}$$

The tachyon equation (2.3) is immediately satisfied by this equation. Let us fix rest of the things from here. Since $F(\infty) = \infty$ we learn from (4.3) that V must be in the vacuum $V(\infty) = 0$ at $T = \infty$, but still $\dot{T} < 1$. More precisely, we can determine from the above

that, as $t \rightarrow \infty$,

$$\frac{V(T)}{\sqrt{1 - \dot{T}^2}} \propto e^{-3\lambda t}.$$

At the time $t = 0$, $V = V_0$, $T(0) = T_0 = 0$, and with the condition $\dot{T}(0) = 0$, we find that

$$H(0) = \lambda \coth\left(\frac{\lambda}{H_0}\right) = \sqrt{\frac{8\pi G}{3} V_0 + \lambda^2}. \quad (4.4)$$

In this way all integration constants are fixed and initial value of Hubble constant is fine tuned to V_0 . Since we are taking $\Lambda \ll 8\pi G V_0$, again only by choosing large V_0 we can start with high value of Hubble constant.

4.1 Vacuum dominated phase

In the tachyon vacuum, where $T \sim \infty$, and $H \sim \lambda$, but still $\dot{T} \neq 1$, the equations of motion will assume following simple form:

$$\begin{aligned} \ddot{T} &= (1 - \dot{T}^2) \left(\frac{1}{\sqrt{2}} - 3\lambda\dot{T} \right) \\ \frac{\dot{a}}{a} &= \lambda \end{aligned} \quad (4.5)$$

where the potential has been $V = V_0/\cosh(T/\sqrt{2})$. This equation suggests that late time evolution of tachyon is not completely decoupled from gravity. The tachyon equation (4.5) can be exactly integrated to give [21]

$$\left(\frac{(1 - \beta\dot{T})^2}{(1 - \dot{T}^2)} \right)^\beta \frac{1 + \dot{T}}{1 - \dot{T}} = e^{\sqrt{2}(1-\beta^2)(t+t_0)} \quad (4.6)$$

where $\beta = 3\sqrt{2}\lambda$ and t_0 is integration constant. Thus, if $\beta < 1$, the $t \rightarrow \infty$ limit of the (4.6) suggests that $(1 - \dot{T}) \rightarrow 0$, that is $T \sim t$ and tachyon ultimately becomes the time. While if $\beta > 1$, the $t \rightarrow \infty$ limit of the (4.6) suggests that $(1 - \beta\dot{T}) \rightarrow 0$, that is $T \sim \frac{t}{\beta}$ and tachyon becomes time only upto a factor. In other words \dot{T} never reaches unity. Specially for $\beta = 1$, i.e. $\lambda = \frac{1}{3\sqrt{2}}$, the integration of the (4.5) gives

$$\frac{1}{1 - \dot{T}} + \ln \sqrt{\frac{1 + \dot{T}}{1 - \dot{T}}} = \sqrt{2}(t + t_0). \quad (4.7)$$

This equation also suggests that $\dot{T} \sim 1$ at late time. Out of these possibilities, $\beta < 1$ case is more favored. In a rather shorter analysis, for $\lambda \ll \frac{1}{3\sqrt{2}}$ the tachyon equation effectively (4.5) becomes

$$\ddot{T} \simeq (1 - \dot{T}^2) \left(\frac{1}{\sqrt{2}} \right)$$

and it has the solution $T \sim \sqrt{2} \ln \cosh\left(\frac{t}{\sqrt{2}}\right)$. Thus ultimately $T \sim t$, i.e. tachyon becomes time and $\dot{T} \rightarrow 1$.

The scale factor corresponding to (4.5) behaves as

$$a(t) \sim e^{\lambda t}$$

which is like in *vacuum dominated inflationary scenario* where the universe accelerates, see [20]. From (4.1), during the evolution where $H(t) \gg \lambda$, the equation reduces to that of $\Lambda = 0$ case, which has a matter dominated decelerating phase characterised by scale factor $a(t) \sim t^{2/3}$. Once $H \sim \lambda$ the vacuum dominated phase takes over where the scale factor behaves as $a(t) \sim e^{\lambda t}$. It means in this model the matter dominated phase will precede the vacuum dominated phase during evolution.

In summary, the universe at $t = 0$ starts with accelerating expansion then enters into decelerating matter dominated intermediate phase and finally exiting into vacuum dominated accelerating phase. In fact from eqn. (4.2) H reaches the terminal value $\lambda = \sqrt{\Lambda/3}$. It is therefore clear that if we want Λ to be the cosmological constant as observed today, then Λ must be fine-tuned in the effective action to the observed value possibly through some stringy mechanism, something like we encounter in KKLТ scenario [18].

4.2 Number of e-folds

The number of e-foldings through which universe expands during the inflation is defined as

$$N_e = \ln \frac{a(t_2)}{a(t_1)} \equiv \int_{t_1}^{t_2} H(t) dt. \tag{4.8}$$

This is an important parameter in cosmology and the estimate is that our universe has gone through close to 60 e-foldings during inflation. We would like to estimate the value of this number in tachyon cosmology. Consider we are at the top of the potential, in this neighborhood the typical value of \dot{T}^2 is infinitesimally small. Then from eq. (4.2), during the inflation Hubble constant behaves as (since $\frac{\lambda}{H_0} \ll 1$)

$$H = \lambda \coth \left[\lambda \left(\frac{3}{2} \int_0^t \dot{T}^2 dt' + H_0^{-1} \right) \right] \simeq \frac{1}{\frac{3}{2} \int_0^t \dot{T}^2 dt' + H_0^{-1}}.$$

Hence the inflation lasts so long as $\frac{3}{2} \int_0^t \dot{T}^2 dt' \ll \frac{1}{H_0}$. The estimate of the number of e-folds during the inflationary interval, τ , can be obtained also by using equation (3.6) which tells us that $\frac{8\pi G V}{3} \leq H^2$ always. We have an important bound

$$N_e = \int_0^\tau H dt \geq \sqrt{\frac{8\pi G}{3}} \int_0^\tau V^{\frac{1}{2}} dt \tag{4.9}$$

In order that we get some estimate of the value of N_e we need to further evaluate the end integral in the inequality (4.9). But it is difficult until we have exact solutions. Note that during inflation both V and H change in time, but from eq.(3.6) it can be determined that H varies less slowly than V . So it can be safely assumed that H to be constant at $H \sim H_0$ though out inflation. We set $H = H_0$ in the integral (4.9) and assuming that inflation ends

at $T(\tau) \simeq 1$ we can evaluate

$$\begin{aligned} \sqrt{\frac{8\pi G}{3}} \int_0^\tau V^{\frac{1}{2}} dt &= \int_{T(0)}^{T(\tau)} \bar{V}^{\frac{1}{2}} \frac{dt}{dT} dT = \int_{T_0}^1 \bar{V}^{\frac{1}{2}} \left(1 - \frac{\bar{V}^2}{H^4}\right)^{-\frac{1}{2}} dT \\ &\approx \int_{T_0}^1 \bar{V}^{\frac{1}{2}} \left(1 - \frac{1}{\cosh^2\left(\frac{T}{\sqrt{2}}\right)}\right)^{-\frac{1}{2}} dT \end{aligned} \quad (4.10)$$

where $\bar{V} \equiv \frac{8\pi G}{3} V$. Making other crude but suitable approximations we estimate the value of the above integral as $\sqrt{2}H_0 \ln \frac{1}{T_0}$, hence

$$N_e \geq \sqrt{2}H_0 \ln \frac{1}{T_0}. \quad (4.11)$$

Typically taking $H_0 = 1$ and $T_0 = 10^{-10}$ in the above, gives us $N_e \geq 32.5$. The value $H_0 \sim 1$ would mean $H_0 \sim O(1)M_p$. However, the form of equation (4.11) suggests that various choices of (H_0, T_0) can give rise to the same value of N_e and so there is flexibility in choosing the initial values. In the next section we will also present a numerical estimates of these quantities.

$\Lambda_{observed}$ and a bound:

In this work we have been interested in taking $\Lambda = \Lambda_{observed} \simeq 10^{-122} M_p^2$. So let us consider the situation where $M_s \ll M_p$. This requires large volume compactification, that is, we must have $\frac{g_s^2}{v_0} \ll 1$ ($g_s \ll 1$). But we need to maintain $H_0^2 = \frac{8\pi G V_0}{3} \gg \Lambda$ at the same time for the rolling tachyon model to work. This restriction translates into the bound

$$1 < \frac{v_0}{g_s^2} \leq 10^{60}.$$

So there is an upper bound on the largeness of the compactification volume given by

$$v_0 = \left(\frac{R}{l_s}\right)^6 \leq g_s^2 10^{60}$$

beyond that our analysis breaks down. This implies there is a lot of freedom in choosing the size of compactification manifold. For example, if we consider an unrealistically large volume compactification saturating the bound $\frac{g_s^2}{v_0} = 10^{-60}$ with weak string coupling, the string scale is pushed close to $M_s \sim 10^{-30} M_p$. But this amounts to bringing down the height of tachyon (inflaton) potential to a low value. So what value of v_0 is optimum and is of phenomenological importance, will depend upon a particular model.

5. Numerical analysis of the tachyon rolling

We mainly aim here to reproduce the numerical results of the tachyon-gravity system for $\Lambda = 0$, while for $\Lambda \neq 0$ the basic results will be similar except at the late time. The two key first order equations are

$$\begin{aligned} \frac{\dot{H}}{H^2} &= -\frac{3}{2}\dot{T}^2 \\ \frac{8\pi G}{3} \frac{V(T)}{\sqrt{1-T^2}} &= H^2. \end{aligned} \quad (5.1)$$

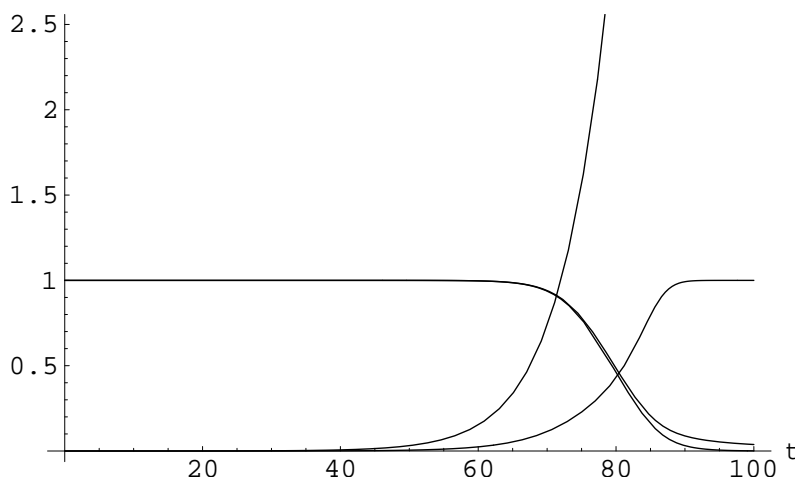


Figure 1: Plots are for the time evolution of T , \dot{T} , H and $\sqrt{8\pi GV/3}$ presented in clockwise manner, with initial values $H(0) = 1$, $T(0) = 10^{-5}$. The value of N_e , which is the area under $H(t)$ curve in the plateau region, could easily be estimated to be close to 60. H vanishes at late time but $V(T)$ vanishes faster than H .

We are interested in the initial conditions where $T(0) = 0$, $H_0 \neq 0$ such that at the top of the potential $\dot{T}(0) = 0$. This implies $\dot{H}(0) = 0$ with $\frac{8\pi G}{3}V(0) = H_0^2$. Classically the system with these initial conditions will not evolve in time, however a small quantum fluctuation will dislocate the configuration from the top and the system will start evolving. We shall take initial conditions in our numerical analysis in conformity with this fact where we are slightly away from the top position. The potential considered in these calculations is $V = V_0/\text{Cosh}(\frac{T}{\sqrt{2}})$. In our units t is measured in M_p^{-1} units and the Hubble constant H is measured in M_p . It must be kept in mind that we have already set $\alpha' = M_s^{-2} = 1$ in the beginning. Now we also set $M_p = 1$, with $\frac{g_s^2}{v_0} = 1$. Note that this has been done primarily for simplification and it will allow us to present the rolling behaviour of the equations (5.1). The results could be exploited in a suitable phenomenological setting.

The numerical results are plotted in figures 1, 2 and 3. The time evolution of the quantities T , \dot{T} , H and $\sqrt{8\pi GV/3}$ could be found in these figures. The plots of H and $\sqrt{8\pi GV/3}$ overlap with each other in the plateau region which characterises the inflationary period. Both H and V vanish at late time, but $V(T)$ vanishes faster than H . This is consistent with the bound we proposed in eq. (4.9). From figure 1, it is found that taking the initial value $H(0) = 1$ and the $T(0) \sim 10^{-5}$ provides us with the number of e-folds during inflation close to 60. The duration of inflation is however very short and is around $60(M_p)^{-1}$. But it can be increased by taking smaller values of $H(0)$ and $T(0)$. Also we observe that increasing the height of the tachyon potential increases the number of e-folds as can be seen by comparing figure 2 and figure 3. From figures 1 and 2 we see that lower is the value of $T(0)$ longer is the duration of inflation which is on the expected lines.

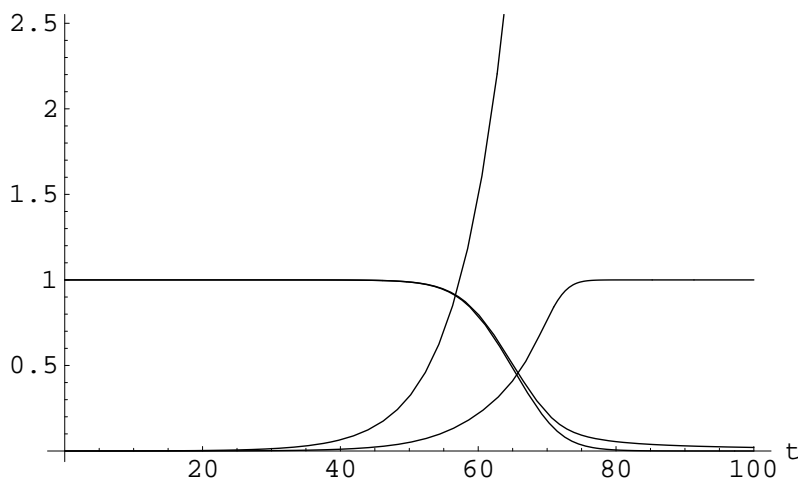


Figure 2: For comparison these plots are for T , \dot{T} , H and $\sqrt{8\pi GV/3}$ but with initial values $H_0 = 1$ and $T(0) = 10^{-4}$. The value of N_e could easily be estimated to be less than 60 in this case. H vanishes at late time but faster than in the previous graph.

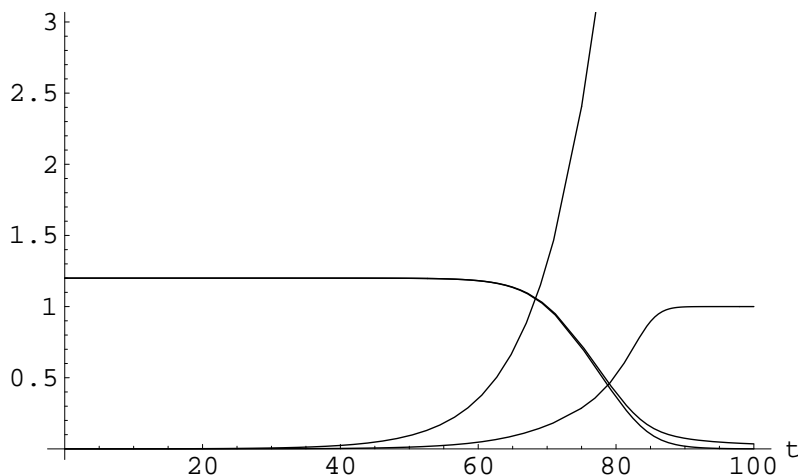


Figure 3: Plots are for the time evolution of T , \dot{T} , H and $\sqrt{8\pi GV/3}$ in clockwise manner, with higher initial value $H_0 = 1.2$, and $T(0)$ same as in second figure. The value of N_e could easily be estimated to be close to 70.

6. Conclusion

We have studied rolling tachyon cosmological solutions in de Sitter space. The Friedman-Robertson-Walker solutions are taken to be spatially flat and those are time-reversal non-symmetric. Only non-singular configurations are allowed as the solutions and rolling tachyon can give rise to the required inflation, which can be obtained by suitably choosing the initial conditions at the top of the tachyon potential. For non-vanishing Λ , the matter dominated decelerating phase, with $a(t) \propto t^{\frac{2}{3}}$, precedes the vacuum dominated accelerat-

ing expansion ($a(t) \propto e^{\lambda t}$) at the end of the evolution. However, we do not encounter the radiation dominated phase, where $a(t) \propto t^{\frac{1}{2}}$. We also find that cosmological constant of the universe has to be fine-tuned at the level of the effective action itself. In which case KKLT model [18] of string compactification can be a good starting point with $\Lambda_{KKLT} = \Lambda_{observed}$, indicating that string dynamics has to be responsible for such a small parameter as cosmological constant today. Interestingly, whatever the initial value of the Hubble constant at $t = 0$, the universe exits in vacuum dominated phase at the end of evolution. We also find that there is an upper bound on the largeness of the compactification volume given as

$$1 < \frac{v_0}{g_s^2} \leq 10^{60}.$$

Although, there are other important issues about cosmological inflation, for example, the amplitudes of CMB density perturbations, reheating of the universe etc. These are beyond the scope of this paper but we hope to address them else where.

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